MATH 113
Linear Algebra
Fall 2020

Challenge Problems 3

Just for fun. You won't be tested on this and you won't get points for it. Feel free to turn them in with your homework or email me or come chat with me about them.

1. Let V and W be two finite-dimensional vector spaces over a field F. Fix a basis for each.

(a) As we saw in class, every linear transformation $T : V \to W$ can be thought of as a graph. If $F = \mathbb{Z}_2$, the nullspace of a linear transformation T has a concise description in terms of its graph. Can you discover it? Can you say anything when $F \neq \mathbb{Z}_2$?

(b) Over any field, the dimension of the image of a linear transformation T has two different concise descriptions in terms of its associated matrix. Can you discover it?

2. Call a real number $\alpha \in \mathbb{R}$ *algebraic* if there exists a polynomial $p(x) \in P(\mathbb{Q})$ so that $p(\alpha) = 0$. Using linear algebra, prove that if α is algebraic and $q(x) \in P(\mathbb{Q})$ is any polynomial with $p(\alpha) \neq 0$ then there exists a polynomial $r(x) \in P(\mathbb{Q})$ such that $\frac{1}{q(\alpha)} = r(\alpha)$.